Ocean Radar Backscatter Statistics and the Generalized Log Normal Distribution

B. L. Gotwols, R. D. Chapman, R. E. Sterner II
Johns Hopkins University, Applied Physics Laboratory

ABSTRACT

A model has been developed which describes the statistics of the radar backscattered signal at mid-incidence angles. The parameters of this model can be derived directly from the backscattered signal itself, without the need for in situ measurements, as in earlier work. This model suggests that sea spikes observed at mid incidence are primarily caused by the highly nonlinear relationship between slope and power. It also suggests a new technique for computing the radar modulation transfer function which has higher coherence than the conventional approach.

Keywords: Radar, Backscatter, Statistics, Log Normal, Sea Spikes, MTF.

1. INTRODUCTION

Recently, a new distribution for the radar backscatter cross section from small footprints at mid-incidence angles was proposed (Ref. 1). A key idea in the development of this new distribution was to write the cross section, \( \sigma_0 \), as an exponential functional of long wave slope component in the radar look direction, \( s_x \). (For simplicity we will henceforth refer to slope component in the radar look direction as simply slope.) Motivated by the modeling results presented in Ref. (2) the exponential was taken as a quadratic in slope: \( \sigma_0 \sim \exp(a_1s_x + a_2s_x^2) \). The slope under discussion here is measured on the scale of the radar footprint and hence may be considered to be approximately normally distributed. When \( a_1 \) equals zero, \( \sigma_0 \) will be log normally distributed. We will refer to the distribution with \( a_1 \) and \( a_2 \) finite as the generalized log normal distribution.

This distribution was compared with dual polarized Ku-band backscatter data taken at 20° and 45° incidence angles and the agreement was found to be excellent. The \( a_1 \) coefficients were computed with a wave-current and electromagnetic scattering simulation code (Ref. 2). This simulation required rather detailed knowledge of the in situ conditions such as the long wave directional spectrum, and an estimate of long wave/short wave interactions (the so called hydrodynamic modulation transfer function). In the current paper we dispense with the need for in situ measurements and simulation calculations and derive the parameters for the generalized log normal distribution from the radar data itself.

2. COMPOUND DISTRIBUTION

We explore in this section a method to compute the probability density of instantaneous amplitude, \( a \), given the short term averaged time series of \( \sigma_0 \). The idea is to treat the ocean surface as a two scale distribution where the short scale is on the order of the radar wavelength, and the long scale is on the order of the antenna footprint (which must be small compared to the dominant ocean waves). We should emphasize that the instantaneous amplitude must be sampled rapidly enough to preserve all of the fluctuations in the received signal due to constructive and destructive interference between the scatterers present in the radar footprint. This is sometimes called fading (Ref. 3). On the other hand, \( \sigma_0 \) represents a short term time average and therefore the fading contribution to \( \sigma_0 \) must be averaged out. Using Bayes conditional probability theorem, the probability density for amplitude can be written as an integral over the product of the conditional amplitude probability density given \( \sigma_0 \), times the probability density of \( \sigma_0 \):

\[
p(a) = \int_0^{\infty} p(a|\sigma_0)p(\sigma_0)d\sigma_0. \quad (1)
\]

This idea, can be traced back at least two decades to the work of Valenzuela and Laing in an unpublished but widely circulated report (Ref. 4). It has been shown (Ref. 1) that at mid-incidence angles, \( p(a|\sigma_0) \) is given by the Rayleigh distribution,

\[
p(a|\sigma_0) = \frac{2a}{\sigma_0}\exp\left(-\frac{a^2}{\sigma_0}\right). \quad (2)
\]
It will be convenient in the development that follows to recast Eq. (1) in terms of the log of $\sigma_0$:

$$l = \ln(\sigma_0). \quad (3)$$

This allows us to work with the log of received power, a great advantage as we shall see below. Eq. (1) may then be rewritten as:

$$p(a) = \int_{-\infty}^{l} p(a|l)p(l)dl. \quad (4)$$

In order to test the validity of Eq. (4) we computed a running time series of 0.25 s estimates of $\sigma_0$ over a 7.5 hour period during which meteorological and oceanographic conditions were reasonably stable. (Details of the parameters that apply to all data analyzed in this paper are summarized in Table I.) After computing these running averages, the log was taken and $p(l)$ estimated. The extremely good agreement between $p(a)$ computed in this manner and the direct measurement of $p(a)$, is shown in Fig. 1. (In order to avoid absolute calibration problems, in this figure we normalized the amplitude by the rms amplitude, $a_0$.) This agreement adds further support to the assumption that $p(a|\sigma_0)$ is Rayleigh distributed as found in Refs. (1 and 5). It also suggests that our choice of a 0.25 s averaging interval is correct for the data set discussed in this paper.

### Table I: Parameters for data discussed in this paper.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Measurement Ht.</td>
<td>47 m</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>4.5 +0.3/-0.2 m/s</td>
</tr>
<tr>
<td>Wind Direction</td>
<td>31° +8°/-9°</td>
</tr>
<tr>
<td>Wind Friction Vel. u*</td>
<td>13.3 cm/s</td>
</tr>
<tr>
<td>Swell Direction</td>
<td>325°</td>
</tr>
<tr>
<td>Radar Look Dir.</td>
<td>0°</td>
</tr>
<tr>
<td>Radar Frequency</td>
<td>14 GHz</td>
</tr>
<tr>
<td>Elapsed Time</td>
<td>7.5 hr</td>
</tr>
<tr>
<td>Acquisition Time</td>
<td>4.0 hr</td>
</tr>
</tbody>
</table>

3. GENERALIZED LOG NORMAL

The results reported in Refs. 1 and 2 showed that $\sigma_0$ can be expressed as a function of long wave slope-component in the radar look direction, $s_x$:

$$\sigma_0 = C \exp(a_1 s_x + a_2 s_x^2). \quad (5)$$

$p(a)$ can be computed by assuming the long-wave slope is normally distributed with standard deviation $\sigma_x$. However, one difficulty with this approach is that it requires knowledge of $\sigma_x$, a quantity which is often lacking in typical radar oceanographic experiments. This difficulty can be overcome if we change from absolute slope-component, $s_x$, to normalized slope component in the radar look direction, $\xi = s_x / \sigma_x$, and rewrite in terms of new model parameters $b_i$:

$$b_1 = a_1 \sigma_x, \quad b_2 = a_2 \sigma_x^2. \quad (6)$$

In addition, redefining $l$ to be zero mean has the advantage that it allows us to eliminate the scaling parameter C, while retaining the relative shape of the probability density function. Thus we have:

$$l = \ln(\sigma_0) - \langle \ln(\sigma_0) \rangle = -b_2 + b_1 \xi + b_2 \xi^2 \quad (7)$$

where we have used the fact that $\xi$ has zero mean and unit variance.

$p(l)$ may now be derived by invoking the transformation law for probability density:

$$p(l)|dl| = p(\xi)|d\xi| \quad (8)$$
which yields

\[ p(l) = \frac{\exp(-\frac{\xi^2}{2})}{\sqrt{2\pi b_1 + 2b_2 \xi^-}} + \frac{\exp(-\frac{\xi^2}{2})}{\sqrt{2\pi b_1 + 2b_2 \xi^+}} \]  \tag{9}

where

\[ \xi\pm = \frac{-b_1 \pm \sqrt{b_1^2 + 4b_2(l + b_1)}}{2b_2} \]  \tag{10}

It should be noted that the positive root in Eq. (9) is not physically realistic since it implies that \( \sigma_0 \) turns up after going through a minimum at some large positive slope (in our sign convention positive corresponds to slopes tilting away from the radar). For the values of \( b_1 \) and \( b_2 \) that we have so far encountered this is not a problem since the values of \( \xi^+ \) correspond to highly improbable slopes.

4. INVERSION

The \( b_n \) parameters may be derived by computing the second and third moments of \( l \) from Eq. (7). The results are:

\[ m_2 = b_1^2 + 2b_2^2 \]  \tag{11}

and

\[ m_3 = (6b_1^2 + 17b_2^2)b_1 \]  \tag{12}

where we have used the convenient moment properties of the normally distributed random variable \( \xi \). Inversion of Eqs. 11 and 12 is a straightforward matter and involves solving a cubic equation for \( b_2 \).

Fig. 2 shows a comparison between the probability density function proposed in Eq. 9 and the measured pdf. \( b_1 \) and \( b_2 \) were computed from the measured pdf following the method discussed above, and are given in Table II. The same runs were analyzed as in Fig. 1. The goodness of fit does not seem to be an exception since it was matched by another run we analyzed where the wind speed was approximately 10 m/s. Further work is needed before we can be confident that the moment matching technique for deriving the \( b_n \) parameters is sufficiently robust to be applied to the large collection of data in our possession.

<table>
<thead>
<tr>
<th></th>
<th>( \text{K}_u)-HH</th>
<th>( \text{K}_u)-VV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>-0.971</td>
<td>-0.745</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.072</td>
<td>0.075</td>
</tr>
</tbody>
</table>

**Table II:** \( b_1 \) and \( b_2 \) parameter values.

![Figure 2: Probability density function of the log of received power. The solid curve is directly estimated from the log of 0.25 s averaged power, while the dashed curve is computed from Eq. (9). (a) \( \text{K}_u \) band, HH polarization, and (b) \( \text{K}_u \) band, VV polarization.](image-url)
5. DISCUSSION

The success of our model at mid incidence suggests to us that many of the so-called sea spikes discussed in the radar literature may be caused by the nonlinear slope to power relation discussed in this paper. Consequently we suggest that sea-spikes are caused by three processes:

1. Fading
2. Nonlinear slope to power transformation
3. Specular and other non-Bragg events due to wave breaking.

Item 1 is unimportant as long as the averaging time is long compared to the correlation time of the received power. Item 2 is probably the most important source of spikes at mid incidence. In fact, one can construct a normally distributed signal and pass it through the nonlinear transformation in Eq. (1) and it will look very spiky, similar to the actual received power. It will be difficult to separate item 3 from the (non-specular) large slope events in item 2. Perhaps an optical technique can be devised to solve this problem.

The exponential slope to power transformation suggests that it may be an improvement to recast the modulation transfer function (MTF) in terms of the log of received power. One could even go one step further and invert all the way back to normalized slope. Preliminary experiments that we have done indicate that either of these approaches noticeably increases the coherence between the linearized quantity and the measured Doppler-inferred currents.

6. CONCLUSIONS

A technique has been developed which allows the statistics of received power at mid-incidence to be modeled without the need for in situ measurements. The success of this model suggests that sea spikes observed at mid incidence may simply be caused by the extremely nonlinear transformation from slope to power. Using the techniques described in this paper it may be possible to derive an improved method for measuring the modulation transfer function.

7. ACKNOWLEDGMENTS

We wish to thank Ken Melville for allowing us to record the data from his K_u-band scatterometer during the SAXON FPN experiment. David Raley wrote the data acquisition program which worked flawlessly during the entire one month experiment. Siegfried Stolte provided meteorological data and Peter Lobemeier provided logistical and archival support. Friedwart Ziemer provided wave directional spectra derived from a roll-pitch buoy. Bill Plant introduced us to the problem of excess noise for small antenna footprints. This work was supported by the U. S. Office of Naval Research.

8. REFERENCES


